Indian Statistical Institute
Bangalore Centre
B. Math. (Hons).
Analysis -II
Mid-semester Examination

March 5, 2009
Time: 3 hours
Maximum marks: 100

(1) Show that for any bounded function on a closed and bounded interval of the real line, lower Riemann integral is less than or equal to the upper Riemann integral. [15]

(2) Compute lower and upper Riemann integrals for the function:

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational;} \\ 0 & \text{otherwise,} \end{cases}$$

on the interval $[0, 1]$. [20]

(3) For $-\infty < a < b < \infty$, show that continuous functions on $[a, b]$ are Riemann integrable. [20]

(4) Define functions h_n on $[0, 1]$ for $n \geq 1$ by

$$h_n(x) = \begin{cases} 1 + nx & \text{if } 0 \leq x \leq \frac{1}{n}; \\ 1 & \text{if } \frac{1}{n} < x \leq 1. \end{cases}$$

Verify as to whether

$$\lim_{n\to\infty} \left(\int_0^1 h_n(x)dx\right) = \int_0^1 \left(\lim_{n\to\infty} h_n(x)\right)dx.$$

[15]

(5) State and prove any one version of the fundamental theorem of calculus. [20]

(6) Define functions g, β on $[-1, 1]$ by

$$g(x) = 3x + 5 \text{ if } -1 \leq x \leq 1,$$

and

$$\beta(x) = \begin{cases} -1 & \text{if } -1 \leq x < 0; \\ 0 & \text{if } x = 0; \\ +1 & \text{if } 0 < x \leq 1. \end{cases}$$

Show that g is Riemann-Stieltjes integrable with respect to β and compute $\int_{-1}^{+1} g(x)d\beta(x)$. [20]